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A NEW CALCULUS.

A First Course in the Differential and Integral Calculus. By Dr. W. F. Osgood. Pp. xv+423; with 125 figures. (New York: The Macmillan Company; London: Macmillan and Co., Ltd., 1907.) Price 10*s.* 6*d.*

THE introduction of the Calculus at an early stage in a course of elementary mathematics has rendered necessary the substitution of simplified methods of treatment for those occurring in the earlier text-books; for example, an abandonment of the lavish and unnecessary use of infinite series, the convergency of which was generally ill-understood, in the differentiation of simple expressions.

A number of good books have recently appeared written more or less with this object in view, but we have seen none in which the survival of old and clumsy methods has been reduced to the vanishing point in the same way that has been done in this book.

The present reviewer has been in the habit of conducting a class in the calculus on simplified lines identical in nearly every respect with those adopted independently by Prof. Osgood; indeed, this book represents almost word for word what he would have wished to write had he undertaken to write a Calculus. The reviewer is thus greatly indebted to the author for having saved him this troublesome and thankless task, or the alternative of continuing the elaborate lecture notes which he has found necessary to dictate to his pupils on the bookwork of the subject. The following is a brief summary of some of the salient features of the book to which the reviewer attaches especial importance.

The methods of the calculus are discussed and exemplified in the first instance by the study of the differentiation of series of positive integral powers only. The reviewer would prefer to see the binomial theorem omitted from the proof for the derivative of x^n and a proof based on the product rule substituted, but this is a minor detail which any teacher can supply for himself and his class.

The very important method of "differentiating an equation as it stands" is explicitly used as such in finding the tangents to algebraic curves as well as in the differentiation of fractional powers, inverse functions, and the like. The introduction of this subject under the title of "differentiation of an implicit function" is quite unnecessary, and we are glad to see that the perfectly simple method really required for these cases is viewed in its right light.

In the chapter on transcendental functions the author clearly points out that the reason for measuring angles in radians is essentially explained by the calculus, and he also gives the differentiation formulæ for angles measured in degrees. The explanation is necessary in order to dispel any doubts the beginner may have previously formed as to the mental sanity of those mathematicians who deliberately chose an incommensurable unit for the measurement of commensurable angles.

The author's introduction of the incommensurable base e, though only one of a number of different possible methods, is even more satisfactory, no previous knowledge being assumed regarding this base, which is shown to make its existence felt as soon as we attempt to differentiate a power of any constant a with respect to its index, or to differentiate the logarithm of the variable to any assumed base a.

When integration is explained the author does not waste too much time in discussing the methods of integrating long and complicated expressions, but proceeds very soon to the consideration of definite integrals and of geometrical and mechanical illustrations.

"Volumes of revolution" only constitute a particular application of a general method of finding the volume of a solid the sections of which parallel to a fixed plane are circles, squares, triangles, or other simple figures. The examples on pp. 159–161 should make this point clear.

Curvature, evolutes, properties of the cycloid, moments of inertia, and attractions are discussed at an early stage. So also are harmonic motion, resisted motion, and damped oscillations.

When infinite series are introduced the student should be ready for the satisfactory and sufficiently rigorous treatment given, especially in connection with convergence.

In dealing with Taylor's theorem, the remainder is carefully attended to, and specially mentioned in connection with the binomial theorem. We should like to have seen the remainder given as a definite integral, but this can readily be supplied in lecture notes.

Partial differentiation is fully discussed, and we notice among the examples the familiar thermodynamic application

$$\frac{dp}{dv}\frac{dv}{dt}\frac{dt}{dp} = -1.$$

There is a useful chapter on solid geometry which introduces the notion of direction cosines, the orthogonal property of confocal conicoids, and the osculating plane of twisted curves.

Double and triple integrals are well discussed. The artifices so commonly used in the older treatment for calculating volumes and centres of gravity by means of single integrations in particular cases had the disadvantage, from which Prof. Osgood's treatment is exempt, of failing to familiarise the student with notions which he necessarily encounters in the study of electricity and other branches of physics.

Hyperbolic functions are not introduced until the end. In the opinion of the reviewer they have figured far too prominently in previous treatments of the calculus, with the result that the student has been encouraged to waste time in working out integrals in complicated forms involving "sneezes" and "coughs," and "sneeze and cough minus ones," which he cannot interpret. It is to be presumed that the above words represent the most natural equivalent in speaking of the new-fashioned hyperbolic notation, for to say

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"essaitch" and "seeaitch" is too cumbersome. For computation integrals ought to be evaluated, as the author does, in the form of logarithms, as there are few students who, when they have obtained any result involving a "sneeze minus one," could calculate its numerical value.

We must not forget to mention the collections of examples, which are of the type approved by the most enlightened examining boards in Great Britain. They are for the most part based on practical applications, and are of such a character as to test the student's knowledge of the *calculus* itself, not his power of covering sheets of foolscap with uncomprehended formulæ.

To sum up, it had become necessary to introduce considerable changes in the elementary treatment of the calculus, not only in the interests of the students of physics and engineering whose claims have been most prominently put forward, but also for the sounder and more rational instruction of mathematical students. The present book admirably meets the requirements of the case. We do not say that further improvements are impossible, but we consider that a stage has now been reached when any attempt to make the treatment better in one particular is very liable to render it worse in another.

May not an analogy also be suggested between the coincidence of the author's and reviewer's views, and probably the views of other teachers, and the conditions in the calculus for maximum value, as showing that the methods adopted are the best possible, subject to present existing conditions.

PREHISTORIC EUROPE.

L'Europe préhistorique. Principes d'Archéologie préhistorique par Sophus Müller, traduit du danois avec la collaboration de l'auteur par Emmanuel Philipot. Pp. 212. (Paris: J. Lamarre, n.d.) Price 10 francs.

THE prehistoric period in Europe is so extended, the conditions during the period so varied according to place and time, our knowledge of the conditions so meagre and broken, that the task of putting the events of the period into the form of a connected narrative is not to be lightly estimated or easily fulfilled.

Dr. Müller decides at the outset to confine himself to the consideration of matters which have received general acceptance; from this resolution he, however, soon departs, or there would have been little to tell.

The work is not so complete as the title implies; the long and important palæolithic phase is summarily dismissed at the foot of the fifteenth page. The author, moreover, has drawn his facts from one source only—that of archæology; the evidence of craniology and philology is ignored. Nor has all the literature been consulted. In a book which deals so much with the prehistoric age of Greece it is strange to find no mention of Prof. Ridgeway and his work. Despite these defects, however, the book forms an interesting and suggestive study; it displays much thought and judgment.

The general argument, which is simple, can be expressed in a few words; it is that in prehistoric time, as in early historic time, Europe was indebted for her culture to Greece and Asia Minor; that the culture extended from the Ægean as a centre, undergoing more and more change as it neared the periphery. Dr. Müller likens south-east Europe, in its relation to the rest of Europe, to a town in its relation to the surrounding country. Just as the habits and culture of a town slowly spread to the rural districts, where they persist and not infrequently attain a greater development than was known in the town, so did the culture of Greece gradually extend over the whole of Europe. While on this analogy it may be well to refer to another feature—sometimes the country misses a step in the development of culture; for instance, in many districts the lamp has been directly superseded by electricity without the intermediate use of gas; so in the north of Europe the Bronze age followed on the heels of the Neolithic, whereas in the south of Europe a Copper age intervened. It will be seen that to establish his argument the author must prove that the different phases of culture appeared earlier in the south than in the north. The higher development of any phase in the north is not against the general trend of his argument. The evidence upon which he grounds his theory is obtained from art objects, polished stone weapons, articles of bronze and copper, pottery, particularly that exhibiting decorative designs, grain, domesticated animals, and the architecture of the graves.

As is well known, culture alone can prove a very misleading guide in correlating people, for where the same environment obtains, there will a similar culture tend to develop. The evidence which is at times admitted cannot be allowed to pass unchallenged. It is gravely argued, for instance, that a correlation existed between the people who lived in Spain and the Pyrenees during the Solutré period and the Iron-age inhabitants of Greece, since statuettes of similar form are forthcoming from both regions. No account is taken of the great difference in the age of the statuettes, a difference to be expressed in thousands of years.

To choose another illustration, the author finds that the polished stone celts are bigger and more numerous in the north than in the south, due to the Stone age enduring longer and attaining a higher development in the north. Moreover, in the north the stone celts are of flint, whereas in the south they are of nephrite, jadeite and chloromelanite, stones rare in Europe but more common in Asia. He thereupon argues that when Man began to polish his stone tools he would use such a soft stone as nephrite or jadeite, and would not begin to polish a stone so hard as flint until the art of polishing had made considerable advance. He therefore concludes that the nephrite celts are earlier than those of flint, and that the art of polishing extended from the south to the north. It may, however, well have been that it was the distribution of the various stones which governed the material of which the celts were made.

He takes again the spiral *motif*—in mid-Europe he

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